# Large "Dipolar" Vacuum Fluctuations in Quantum Gravity

Giovanni Modanese[1]

*California Institute for Physics and Astrophysics*
*366 Cambridge Ave., Palo Alto, CA 94306*

and

*University of Bolzano – Industrial Engineering*
*Via Sorrento 20, 39100 Bolzano, Italy*

**Abstract**

We study a novel set of gravitational field configurations, called "dipolar zero modes", which give an exactly null contribution to the Einstein action and are thus candidates to become large fluctuations in the quantized theory. They are generated by static unphysical sources satisfying (up to terms of order $G^2$) the simple condition $\int d^3x T_{00}(\mathbf{x}) = 0$. We give two explicit examples of virtual sources: (i) a "mass dipole" consisting of two separated mass distributions with different signs; (ii) two concentric "+/- shells". The field fluctuations can be large even at macroscopic scale. There are some, for instance, which last $\sim 1\ s$ or more and correspond to the field generated by a virtual source with size $\sim 1\ cm$ and mass $\sim 10^6\ g$. This appears paradoxical, for several reasons, both theoretical and phenomenological. We also give an estimate of possible suppression effects following the addition to the pure Einstein action of cosmological or $R^2$ terms.

04.20.-q Classical general relativity.
04.60.-m Quantum gravity.

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## 1 Introduction

Vacuum fluctuations are an essential ingredient of any quantum field theory, and also in quantum gravity they play an important role. The presence in the gravitational action of a dimensional coupling of the order of $10^{-33}\ cm$ – the "Planck length" – indicates that the strongest fluctuations occur at very small scale: this is the famous "spacetime foam", first studied by Wheeler and then by Hawking and Coleman through functional integral techniques [1].

More recently, Ashtekar and others [2] analysed the possible occurrence of large fluctuations in 2+1 gravity coupled to matter. In this case the theory is classically solvable and admits a standard Fock-space quantization. In 3+1 dimensions, however, Einstein quantum gravity is a

[1]e-mail address: giovanni.modanese@unibz.it

notoriously intractable theory, where everything (states, transition amplitudes, time...) is highly non-trivial.

The non-renormalizable UV divergences of the perturbative expansion may indicate that quantum gravity is not a fundamental microscopic theory, but an effective low-energy limit [3], and will be eventually replaced by a theory of strings or branes. On the other hand, it is known from particle physics that the Einstein lagrangian can be obtained, without any geometrodynamical assumption, as the only one which correctly accounts for a gravitational force mediated by helicity-2 particles [4]. For this reason, it is important to investigate – besides the standard perturbative expansion – all the basic properties of the Einstein lagrangian. In the past years we took an interest into Wilson loops [5], vacuum correlations at geodesic distance [6], and the expression of the static potential through correlations between particles worldlines [7].

In this work we study a set of gravitational field configurations, called "dipolar zero modes", which were not considered earlier in the literature. They give an exactly null contribution to the Einstein action, being thus candidates to become large fluctuations in the quantized theory. We give an explicit expression, to leading order in $G$, for some of the field configurations of this (actually quite large) set. We also give an estimate of possible suppression effects following the addition to the pure Einstein action of cosmological or $R^2$ terms.

Our zero modes have two peculiar features, which make them relatively easy to compute: (i) they are formally solutions of the Einstein equations with auxiliary virtual sources; (ii) their typical length scale is such that they can be treated in the weak field approximation. We shall see that these fluctuations can be large even on a "macroscopic" scale. There are some, for instance, which last $\sim 1\ s$ or more and correspond to the field generated by a virtual source with size $\sim 1\ cm$ and mass $\sim 10^6\ g$. This seems paradoxical, for several reasons, both theoretical and phenomenological. We have therefore been looking for possible suppression processes. Our conclusion is that a vacuum energy term $(\Lambda/8\pi G)\int d^4x\sqrt{g(x)}$ in the action could do the job, provided it was scale-dependent and larger, at laboratory scale, than its observed cosmological value. This is at present only a speculative hypothesis, however.

The dipolar fluctuations owe their existence to the fact that the pure Einstein lagrangian $(1/8\pi G)\sqrt{g(x)}R(x)$ has indefinite sign also for static fields. It is well known that the non-positivity of the Einstein action makes an Euclidean formulation of quantum gravity difficult; in that context, however, the "dangerous" field configurations have small-scale variations and could be eliminated, for instance, by some UV cut-off. This is not the case of the dipolar zero modes. They exist at any scale and do not make the Euclidean action unbounded from below, but have instead null (or $\ll \hbar$) action.

A static virtual source will generate a zero mode provided it satisfies the condition $\int d^3x T_{00}(\mathbf{x}) = 0$ up to terms of order $G^2$. The cancellation of the terms of order $G$ (Section 2.2) is important from the practical point of view. In our earlier work on dipolar fluctuations [8] we developed some general remarks based on the form of Einstein equations, and the result was that in order to generate a zero mode the positive and negative masses of the source should differ by a quantity of order $G$, namely $\sim Gm^2/r \sim mr_{Schw.}/r$; this is very small for weak fields, but sufficient to produce a "monopolar" component which complicates the situation. Explicit calculations in Feynman gauge now have shown that the terms of order $G$ cancel out exactly. This opens the way to an amusing "virtual source engineering" work, to find explicitly some zero modes and give quantitative estimates in specific cases.

When analysing the Wilson loops, we had already pointed out some differences in the behavior of gravity and ordinary gauge theories, essentially due to the different signs of the allowed physical sources. Here, again, these differences are apparent. In gauge theories the real sources can be both positive and negative; therefore one can close two Wilson lines at infinity and find the static potential. The virtual sources cannot give rise to strong static dipolar fluctuations, because the lagrangian is quadratic in the fields. On the contrary, in gravity there are no real negative sources, the potential is always attractive and Wilson lines cannot be closed; however, since the lagrangian on-shell is indefinite in sign and equal to $\sqrt{g(x)}\mathrm{Tr}T(x)$, we can construct static zero modes employing +/- virtual sources. Then, of course, we can Lorentz-boost these modes in all possible ways.

The paper is composed of two main Sections. Section 2 is devoted to the analysis of the dipolar fields and virtual sources. We start from some general features and then focus on two examples. Section 3 contains an extensive discussion. For a summary of the main contents see also the *Conclusions* Section.

### 1.1 Conventions. Sign of $\Lambda$ vs. its classical effects

Let us define here our conventions. We consider a gravitational field in the standard metric formalism; the action includes possibly a cosmological term:

$$S = S_{Einstein} + S_\Lambda \tag{1}$$

$$S_{Einstein} = -\frac{1}{16\pi G}\int d^4x\sqrt{g(x)}R(x) \tag{2}$$

$$S_\Lambda = \frac{\Lambda}{8\pi G}\int d^4x\sqrt{g(x)} \tag{3}$$

with $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$.

By varying this action with respect to $\delta g_{\mu\nu}(x)$ and using the relation

$$\frac{\delta\sqrt{g}}{\delta g_{\mu\nu}} = \frac{1}{2}\sqrt{g}g^{\mu\nu} \tag{4}$$

one finds the field equations

$$R_{\mu\nu}(x) - \frac{1}{2}g_{\mu\nu}(x)R(x) + \Lambda g_{\mu\nu}(x) = -8\pi G T_{\mu\nu}(x) \tag{5}$$

The energy-momentum tensor of a perfect fluid has the form

$$[T_{\mu\nu}] = \mathrm{diag}(\rho, p, p, p) \tag{6}$$

For a zero-pressure "dust" one has $p = 0$.

Now let us introduce a signature for the metric. Articles in General Relativity or cosmology use most often the metric with signature $(-,+,+,+)$, and the experimental estimates of $\Lambda$ are mainly referred to this metric. It is important to fix the sign of the cosmological term with reference to the metric signature in a way which is clear both formally and intuitively.

If spacetime is nearly flat, we can take the cosmological term in (5) to the r.h.s., set $g_{\mu\nu}(x) = \eta_{\mu\nu}$ and regard it as a part of the source. We obtain, in matrix form

$$\left[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right] = -\left\{\mathrm{diag}(-\Lambda, \Lambda, \Lambda, \Lambda) + 8\pi G\,\mathrm{diag}(\rho, p, p, p)\right\} \qquad [\text{metric } (-,+,+,+)] \tag{7}$$

Which sign for $\Lambda$ allows to obtain a static solution? Even without finding explicitly this solution, we see that for $\Lambda > 0$ the "pressure" due to the cosmological term is positive and can sustain the system against gravitational collapse – especially in the case of a zero-pressure dust with $p = 0$.

At the same time, the mass-energy density due to the cosmological term is negative and subtracts from $\rho$, still opposing to the collapse.

In conclusion, with this convention on the metric signature a static solution of Einstein equations with a cosmological term can be obtained for $\Lambda > 0$. If we are not interested into a static solution, but into an expanding space à la Friedman-Walker, in that case the effect of a cosmological term with $\Lambda > 0$ will be that of accelerating the expansion. The most recent measurements of the Hubble constant from Type Ia supernovae [9, 10] suggest indeed that there is a cosmologically significant positive $\Lambda$ in our universe.

In Quantum Field Theory, on the other hand, the signature $(+,-,-,-)$ is more popular, such that the squared four-interval is $x^2 = t^2 - |\mathbf{x}|^{\mathbf{2}}$. Since we shall introduce some coupling of gravity to matter fields in the following, and make a correspondence to the Euclidean case, we prefer to use this latter convention. We then have, instead of eq. (7)

$$\left[R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right] = -\left\{\mathrm{diag}(\Lambda, -\Lambda, -\Lambda, -\Lambda) + 8\pi G \mathrm{diag}(\rho, p, p, p)\right\} \qquad [\text{metric } (+,-,-,-)] \quad (8)$$

In this case, a static solution – or an accelerated expansion – corresponds to $\Lambda < 0$.

## 2 The dipolar fluctuations

We consider the functional integral of pure quantum gravity, which represents a sum over all possible field configurations weighed with the factor $\exp[i\hbar S_{Einstein}]$ and possibly with a factor due to the integration measure. The Minkowski space is a stationary point of the vacuum action and has maximum probability. "Off-shell" configurations, which are not solutions of the vacuum Einstein equations, are admitted in the functional integration but are strongly suppressed by the oscillations of the exponential factor.

Due to the appearance of the dimensional constant $G$ in the Einstein action, the most probable quantum fluctuations of the gravitational field "grow" at very short distances, of the order of $L_{Planck} = \sqrt{G\hbar/c^3} \sim 10^{-33}\ cm$. This led Hawking, Coleman and others to depict spacetime at the Planck scale as a "quantum foam" [1], with high curvature and variable topology. For a simple estimate (disregarding of course the possibility of topology changes, virtual black holes nucleation etc.), suppose we start with a flat configuration, and then a curvature fluctuation appears in a region of size $d$. How much can the fluctuation grow before it is suppressed by the oscillating factor $\exp[iS]$? (We set $\hbar = 1$ and $c = 1$ in the following.) A naive dimensional estimate suggests that $|R|$ should not exceed $\sim G/d^4$, but in fact only a non-perturbative calculation can give reliable results in the short-distance regime. The most accurate estimates of the critical exponents in lattice quantum gravity are those obtained by Hamber through the Euclidean Regge calculus [11], and show that the correct behavior in four dimensions is

$$|R| \sim \frac{1}{L_{Planck} d} \qquad (9)$$

This is a consequence of the fact that the critical exponent $\nu$, related to the derivative of the gravitational $\beta$-function in the vicinity of the UV fixed point, is very close to 1/3.

### 2.1 General features

There is another way, however, to obtain vacuum field configurations with action smaller than 1 in natural units. This is due to the fact that the Einstein action has indefinite sign. Consider the

Einstein equations with a source $T_{\mu\nu}(x)$

$$R_{\mu\nu}(x) - \frac{1}{2} g_{\mu\nu}(x) R(x) = -8\pi G T_{\mu\nu}(x) \tag{10}$$

and their covariant trace

$$R(x) = 8\pi G \mathrm{Tr} T(x) = 8\pi G g^{\mu\nu}(x) T_{\mu\nu}(x) \tag{11}$$

Let us consider a solution $g_{\mu\nu}(x)$ of equation (10) with a source $T_{\mu\nu}(x)$ obeying the additional integral condition

$$\int d^4x \sqrt{g(x)} \mathrm{Tr} T(x) = 0 \tag{12}$$

Taking into account eq. (11) we see that the pure Einstein action (2) computed for this solution is zero. Thus the tensor $T_{\mu\nu}(x)$ only serves as an auxiliary source in order to construct zero-modes for the action of pure gravity. Condition (12) can be satisfied by energy-momentum tensors that are not identically zero, provided they have a balance of negative and positive signs, such that their total integral is zero. Of course, they do not represent any acceptable physical source, but the corresponding solutions of (10) exist nonetheless, and are zero modes of the pure Einstein action.

We shall give two explicit examples of auxiliary sources (we shall call them "virtual sources" in the following, because they generate virtual field configurations): (i) a "mass dipole" consisting of two separated mass distributions with different signs; (ii) two concentric "+/- shells". In both cases there are some parameters of the source which can be varied: the total positive and negative masses $m_\pm$, their distance, the spatial extension of the sources.

The procedure for the construction of the zero mode corresponding to the dipole is the following. One first considers Einstein equations with the virtual source without fixing the parameters yet. Then one solves them with a suitable method, for instance in the weak field approximation when appropriate. Finally, knowing $g_{\mu\nu}(x)$ one adjusts the parameters in such a way that condition (12) is satisfied.

## 2.2 Computation of $\sqrt{g(x)} g^{00}(x)$

Now suppose we have a suitable virtual source, with some free parameters, and we want to adjust them in such a way to generate a zero-mode $g_{\mu\nu}(x)$ for which $S_{Einstein}[g] = 0$. We shall always consider static sources where only the component $T_{00}$ is non vanishing. The action of their field is

$$S_{zero-mode} = -\frac{1}{2} \int d^4x \sqrt{g(x)} g^{00}(x) T_{00}(x) \tag{13}$$

To first order in $G$, the field $h_{\mu\nu}(x)$ generated by a given mass-energy distribution $T_{\mu\nu}(x)$ is given by an integral of the field propagator $P_{\mu\nu\rho\sigma}(x, y)$ over the source:

$$h_{\mu\nu}(x) = \int d^4y P_{\mu\nu\rho\sigma}(x, y) T^{\rho\sigma}(y) \tag{14}$$

where in Feynman gauge $P_{\mu\nu\rho\sigma}(x, y)$ is given, with our conventions on the metric signature, by

$$P_{\mu\nu\rho\sigma}(x, y) = \frac{2G}{\pi} \frac{\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}}{(x-y)^2 + i\varepsilon} \tag{15}$$

Computing the integral over time in eq. (14) we obtain for our source

$$
\begin{aligned}
h_{\mu\nu}(\mathbf{x}) &= \int_{-\infty}^{+\infty} dy_0 \int d^3y T^{00}(\mathbf{y}) P_{\mu\nu 00}(x,y) \\
&= \frac{2G}{\pi}(2\eta_{\mu 0}\eta_{\nu 0} - \eta_{\mu\nu}\eta_{00}) \int_{-\infty}^{+\infty} dy_0 \int d^3y \frac{T^{00}(\mathbf{y})}{(x_0-y_0)^2 - (\mathbf{x}-\mathbf{y})^2 + i\varepsilon} \\
&= 2G(2\eta_{\mu 0}\eta_{\nu 0} - \eta_{\mu\nu}\eta_{00}) \int d^3y \frac{T^{00}(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|} \qquad (16)
\end{aligned}
$$

Thus we have

$$
\begin{aligned}
\sqrt{g(x)}g^{00}(x) &= \left[1 + \frac{1}{2}\mathrm{Tr}h(\mathbf{x}) + o(G^2)\right]\left[1 + h^{00}(\mathbf{x}) + o(G^2)\right] \\
&= 1 + \frac{1}{2}\mathrm{Tr}h(\mathbf{x}) + h^{00}(\mathbf{x}) + o(G^2) \\
&= 1 + 2G\left[\frac{1}{2}(2\eta_{\mu 0}\eta_{\nu 0} - \eta_{\mu\nu}\eta_{00})\eta^{\mu\nu} + (\eta_{00})^2\right] \int d^3y \frac{T^{00}(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|} + o(G^2) \\
&= 1 + o(G^2) \qquad (17)
\end{aligned}
$$

and finally the action is

$$
S_{zero-mode} = -\frac{1}{2}\int d^4x T_{00}(x) + o(G^2) \qquad (18)
$$

Therefore provided the integral of the mass-energy density vanishes, the action of our field configuration is of order $G^2$, i.e., practically negligible, as we check now with a numerical example. Let us choose the typical parameters of the source as follows:

$$
\begin{aligned}
r &\sim 1\ cm \\
m &\sim 10^k\ g \simeq 10^{37+k}\ cm^{-1} \qquad (19)
\end{aligned}
$$

(implying $r_{Schw.}/r \sim 10^{-29+k}$). We assume in general an adiabatic switch-on/off of the source, thus the time integral contributes to the action a factor $\tau$. We shall keep $\tau$ (in natural units) very large, in order to preserve the static character of the field. Here, for instance, let us take $\tau \sim 1\ s \simeq 3 \cdot 10^{10}\ cm$. With these parameters we have

$$
S_{zero-mode}^{order\ G^2} \sim \tau \frac{G^2 m_\pm^2}{r^3} \sim 10^{-20+3k} \qquad (20)
$$

Thus the field generated by a virtual source with typical size (19), satisfying the condition $\int d^3x T^{00}(\mathbf{x}) = 0$, has negligible action even with $k = 6$ (corresponding to apparent matter fluctuations with a density of $10^6\ g/cm^3$ !) This should be compared to the huge action of the field of a *single*, unbalanced virtual mass $m$; with the same values we have

$$
S_{single\ m} = -\frac{1}{16\pi G}\int d^4x \sqrt{g(x)}R(x) = -\frac{1}{2}\int d^4x\sqrt{g(x)}\mathrm{Tr}T(x) \sim \frac{1}{2}\tau m + o(G^2) \sim 10^{47+k} \quad (21)
$$

This example shows that the cancellation of the first order term in (17) allows to obtain a simple lower bound on the strength of the fluctuations. In principle, however, one could always find all the terms in the classical weak field expansion, proportional to $G$, $G^2$, $G^3$, etc., and adjust $T_{00}$ as to have $S_{zero-mode} = 0$ exactly. They can be represented by those Feynman diagrams of perturbative quantum gravity which contain vertices with 3, 4 ... gravitons but do not contain any loops. The ratio between each contribution to $S$ and that of lower order in $G$ has typical magnitude $r_{Schw.}/r$, where $r_{Schw.} = 2\pi G m_\pm$ is the Schwarzschild radius corresponding to one of the two masses and $r$ is the typical size of the source. For a wide range of parameters, this ratio is very small, so the expansion converges quickly. From now on we agree that the "$o(G^2)$" term in eq. (18) comprises all the terms quadratic in the field, like for instance that arising from the expansion of $\sqrt{g(x)}$.

### 2.3 Explicit examples of static virtual sources

*(i) The mass dipole*

As an example of unphysical source which satisfies (12) one can consider the static field produced by a "mass dipole". Certainly negative masses do not exist in nature; here we are interested just in the formal solution of (10) with a suitable $T_{\mu\nu}$, because for this solution we have $\int d^4x\sqrt{g}R = 0$. Let us take the following $T_{\mu\nu}$ of a static dipole centered at the origin ($m_+, m_- > 0$):

$$T_{\mu\nu}(\mathbf{x}) = \delta_{\mu 0}\delta_{\nu 0}\left[\frac{m_+}{r_+^3}f_+(\mathbf{x}) - \frac{m_-}{r_-^3}f_-(\mathbf{x})\right] \tag{22}$$

where

$$f_\pm(\mathbf{x}) \equiv f\left(\frac{\mathbf{x} \pm \mathbf{a}}{r_\pm}\right) \tag{23}$$

and $f(\mathbf{x})$ is a smooth test function with range $\sim 1$ and normalized to 1, which represents the mass density. Thus we have a positive source of mass $m_+$ and radius $r_+$ (placed at $\mathbf{x} = -\mathbf{a}$) and a negative source with mass $-m_-$ and radius $r_-$ (placed at $\mathbf{x} = \mathbf{a}$). The radii of the two sources are such that $a \gg r_\pm \gg r_{Schw.}$, where $r_{Schw.}$ is the Schwarzschild radius corresponding to the mass $m_+$.

The mass $m_-$ is in general slightly different from $m_+$ and chosen in such a way to compensate the small difference, due to the $\sqrt{g}g^{00}$ factor, between the integrals

$$I_+ = \int d^4x\sqrt{g(x)}g^{00}(x)\frac{f_+(\mathbf{x})}{r_+^3} \qquad \text{and} \qquad I_- = \int d^4x\sqrt{g(x)}g^{00}(x)\frac{f_-(\mathbf{x})}{r_-^3} \tag{24}$$

The action of the dipole is

$$S_{Dipole} = -\frac{1}{2}\int d^4x T_{00}(\mathbf{x}) = -\frac{1}{2}\tau(m_+ - m_-) + o(G^2) \tag{25}$$

The condition for $S_{Dipole} = 0$ is $m_+ = m_-$, apart from terms of order $G^2$ (i.e., our dipoles have in reality a tiny monopolar component).

Also note that the values of the masses and the radii $r_\pm$ (both of order $r$) can vary in a continuous way – under the only condition that $m_+ = m_-$. This implies that these (non singular) "dipolar" fields constitute a subset with nonzero volume in the functional integration. Actually, they are only a subset of the larger class of solutions of the Einstein equations with sources satisfying eq. (12).

*(ii) The concentric +/- shells*

Consider two concentric spherical shells in contact, the internal one with radii $r_1$, $r_2$, and the external one with radii $r_2$, $r_3$ ($r_1 < r_2 < r_3$). Let the internal shell have mass density $\rho_1$ and the external shell density $\rho_2$, with opposite sign. The condition for zero action requires, up to terms of order $G^2$, that the total positive mass equals the total negative mass, i.e.,

$$\rho_1(r_2^3 - r_1^3) + \rho_2(r_3^3 - r_2^3) = 0 \tag{26}$$

(more generally, if the densities $\rho_1$ and $\rho_2$ are not constant throughout the shells, one has a suitable integral condition).

The spherical symmetry of the corresponding field configuration offers some advantages when one computes the contributions to the cosmological term and the Newtonian self energy (compare Sect. 3.1).

### 2.4 Contribution of virtual dipoles to the cosmological and $R^2$ terms

In the previous Sections we have seen that the pure Einstein action admits zero-modes having the form of virtual dipole field configurations with a small monopole residual. These field configurations are characterized by the parameters $r_\pm$ (radii of the virtual +/- sources), $a$ (distance between the sources) and $m_\pm$ (masses of the sources). We worked out these configurations as solutions of the linearized Einstein equations. We also checked that the weak field approximation is appropriate in a whole "macroscopic" range of the parameters $r_\pm$, $a$ and $m$. This is possible because these configurations (unlike the spacetime foam at the Planck scale) yield $\int d^4x\sqrt{g(x)}R(x) = 0$ thanks to a cancellation between the $R$ contributions in two distinct regions of space. Similar considerations can be done for the field of the concentric +/- shells.

It is natural to ask whether the dipolar fluctuations can be suppressed by other terms present in the gravitational action besides the pure Einstein term. Possible candidates are the $R^2$ terms (usually relevant, however, only at very small distance) and the cosmological term. Let us first look at the latter (see also our general remarks on the role of a cosmological constant in quantum gravity in Section 3.2).

When a static source is spherically symmetric, we can use *outside* it the exact Schwarzschild metric with invariant interval

$$ds^2 = \left(1 - \frac{2GM}{r}\right)^{-1} dt^2 - \left[\left(1 - \frac{2GM}{r}\right) dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{27}$$

The determinant of this metric equals that of flat space, so the presence of one single spherically-symmetric source does not change the volume of the outer space and does not contribute to the cosmological term.

We shall therefore handle separately the cases of the mass dipole and the concentric +/- shells.

*The mass dipole*

In the linearized approximation the integral $S_\Lambda = (\Lambda/8\pi G)\int d^4x\sqrt{g(x)}$ for a dipolar fluctuation can be splitted into the sum of the integrals of the field $h_+(\mathbf{x})$ generated by the positive mass and the field $h_-(\mathbf{x})$ generated by the negative mass. Both fields are spherically symmetric, thus there is no contribution of order $G$ to $S_\Lambda$ outside the sources.

To order $h^2 \sim (Gm)^2$ the field outside the sources differs from the sum of their Schwarzschild fields, and we do have some contributions to the cosmological term, but they are very small. One finds, inserting the numerical values (19) and the current estimate for $|\Lambda|G$, namely $|\Lambda|G \sim 10^{-116}$

$$\Delta S_{\Lambda,outside} \sim \tau^2|\Lambda|Gm^2 \sim 10^{-22+2k} \tag{28}$$

On the other hand, the integrals of $\sqrt{g(x)}$ *inside* the sources contribute to the action already at first order in $h_{\mu\nu}$. Let us use the explicit solutions in Feynman gauge found in the previous section and disregard the effect of the positive source inside the negative one and viceversa. (This will give small corrections proportional to $a/r_\pm$, but does not change the magnitude orders.) We denote by $\omega(\mathbf{x})$ the characteristic function of a 3-sphere with unit radius placed at the origin of the coordinates, and define

$$\omega_\pm(\mathbf{x}) \equiv \omega\left(\frac{\mathbf{x}\pm\mathbf{a}}{r_\pm}\right) \tag{29}$$

We then have, to leading order

$$\begin{aligned}\Delta S_{\Lambda,inside} &= \frac{\Lambda}{8\pi G}\left[\int d^4x\frac{1}{2}\mathrm{Tr}h_+(\mathbf{x})\omega_+(\mathbf{x}) + \int d^4x\frac{1}{2}\mathrm{Tr}h_-(\mathbf{x})\omega_-(\mathbf{x})\right] \\ &= \frac{\Lambda}{8\pi G}\frac{\tau}{2}\int d^3x\omega_+(\mathbf{x})\left(\frac{-4m_+G}{r_+^3}\right)\int d^3y\frac{f_+(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|} + \\ &+\frac{\Lambda}{8\pi G}\frac{\tau}{2}\int d^3x\omega_+(\mathbf{x})\left(\frac{4m_-G}{r_-^3}\right)\int d^3y\frac{f_-(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}\end{aligned} \tag{30}$$

In the double integrals we can suitably shift the variables by $\pm\mathbf{a}$ and re-scale them as $\mathbf{x}\to\mathbf{x}'r_\pm$, $\mathbf{y}\to\mathbf{y}'r_\pm$, obtaining a pure number $\xi$ of order 1 multiplied by $r_+^5$ and $r_-^5$, respectively. Finally we obtain

$$\Delta S_{\Lambda,inside} = -\frac{\xi}{4\pi}\tau\Lambda(m_+r_+^2 - m_-r_-^2) \tag{31}$$

with

$$\xi = \int d^3x'\int d^3y'\frac{\omega(\mathbf{x}')f(\mathbf{y}')}{|\mathbf{x}'-\mathbf{y}'|} \tag{32}$$

With the usual values we find, apart from an adimensional constant of order 1

$$\Delta S_{\Lambda,inside} \sim 10^{-3+k} \tag{33}$$

This means that a relatively small increase in the value of $|\Lambda|$ would be sufficient to suppress the strongest fluctuations (except for those with $r_+ = r_-$ exactly).

*The concentric +/- shells*

In this case $\Delta S_{\Lambda,outside}$ vanishes exactly. Inside the source we have to leading order

$$\Delta S_{\Lambda,inside} = \frac{\Lambda\tau}{8\pi G}\frac{1}{2}\int d^3x\mathrm{Tr}h(\mathbf{x}) \tag{34}$$

Since $\mathrm{Tr}h(\mathbf{x}) = 4V_{Newt.}(\mathbf{x})$ (compare eq. (16)), the integral is a special case of one we shall compute in Section 3.1 The result is

$$\Delta S_{\Lambda,inside} = \Lambda\tau m r^2 Q(\beta) \tag{35}$$

where $r_2 \equiv r$, $r_3 \equiv \beta r$ and $Q(\beta)$ is an adimensional polynomial which can be either positive or negative, depending on the ratio $|\rho_1|/|\rho_2|$. The magnitude order is the same as for the dipole.

Finally, a word about the $R^2$ term. It is typically of the form

$$S_{R^2} = \alpha\int d^4x\sqrt{g(x)}R^2(x) \tag{36}$$

where $\alpha$ is a (small) adimensional coupling and $R^2$ can be replaced by more complex scalars like $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ etc. For an order of magnitude estimate it suffices to multiply the square of the curvature in the sources, namely $R^2 \sim (G\mathrm{Tr}T)^2 = (Gm/r^3)^2$ by their volume $V^{(4)} \sim \tau r^3$. We find in this way, still with the same parameters,

$$S_{R^2} \sim \alpha\tau G^2 \frac{m^2}{r^3} \sim \alpha 10^{-48+2k} \tag{37}$$

Thus the allowed values for $m$ are very large, i.e., there is no significant suppression of the virtual dipoles by the $R^2$ terms at this scale.

# 3 Discussion

## 3.1 Why are these fluctuations paradoxical

The order of magnitude estimates given in the previous Section show that the dipolar vacuum fluctuations allowed in the functional integral formulation of pure Einstein quantum gravity (i.e., such to give $S \ll 1$ in natural units) are very intense also at macroscopic scale.

One may think that such large fluctuations, if real, would not remain unnoticed. Even though vacuum fluctuations are homogeneous, isotropic and Lorentz-invariant, they could manifest themselves as noise of some kind. Most authors are skeptic about the possibility of detecting the noise due to spacetime foam [12, 13], but the virtual dipole fluctuations described in this paper are much closer to the laboratory scale. Observable quantities, like for instance the invariant intervals $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ and the connection coefficients $\Gamma^\rho_{\mu\nu}$ could then exhibit strong fluctuations.

The existence of these fluctuations would be paradoxical, however, already at the purely conceptual level. Common wisdom in particle physics states that the vacuum fluctuations in free space correspond to virtual particles or intermediate states which live very short, i.e., whose lifetime is close to the *minimum* allowed by the Heisenberg indetermination relation. Let us first give a brief formal justification of this rule, and then compare it to our dipole fluctuations.

It is often the case that a quantum field theory has an imaginary time formulation, where the (positive-definite) lagrangian density corresponds to the original hamiltonian density $H$. For a scalar field, for instance, one has $H = (1/2)[(\partial\phi)^2 + (\mathrm{grad}\phi)^2 + m^2\phi^2]$ and the Euclidean functional integral is given by $z_{Eucl} = \int d[\phi] \exp[-\int dt \int d^3x H(t, \mathbf{x})]$. A field fluctuation localized to a region of size $\tau V^{(3)}$ is weighed in the functional integral by the factor $\exp[-\tau V^{(3)} H] = \exp[-\tau E]$ and is thus effectively suppressed unless approx. $\tau E < 1$. Another notable example is the electromagnetic field. Also in this case the analytical continuation of the lagrangian $L = (-1/8\pi)[\mathbf{E}^2 - \mathbf{B}^2]$ yields the energy density $H = (1/8\pi)[\mathbf{E}^2 + \mathbf{B}^2]$; to check this, one just needs to impose the $A_0 = 0$ gauge and remember that only the electric field contains time derivatives of $\mathbf{A}$.

Now let us estimate the product $E\tau$ for the dipolar fluctuations. The total energy of a static gravitational field configuration vanishing at infinity is the ADM energy. Since the source of a dipolar fluctuation satisfies the condition $\int d^3x T_{00}(\mathbf{x}) = 0$ up to terms of order $G^2$, the dominant contribution in the ADM energy is the Newtonian binding energy [14].

The binding energy of the field generated by a source of mass $m$ and size $r$ is of the order of $E \sim -Gm^2/r$, where the exact proportionality factor depends on the details of the mass distribution. For a dipolar field configuration characterized by masses $m_+$ and $m_-$ and radii of the sources $r_+$ and $r_-$, the total gravitational energy is of the order of

$$E_{tot} \sim -Gm_{\pm}^2 \left( \frac{1}{r_-} + \frac{1}{r_+} \right) \tag{38}$$

(disregarding the interaction energy between the two sources, proportional to $1/a \ll 1/r$). For an order of magnitude estimate with the parameters (19) we can suppose that $r_+$ and $r_-$ are both of the order of 1 $cm$. We then have $E_{tot} \sim Gm_{\pm}^2 \sim 10^{12+k}\ cm^{-1}$. Remembering that $k$ can take values up to $k = 6$, we find for these dipolar fluctuations $\tau E_{tot} \sim 10^{28}$!

(For comparison, remember the case of a "monopole" fluctuation of virtual mass $m$ and duration $\tau$. The condition $S < 1$ implies $\tau m < 1$. The dominant contribution to the ADM energy is just $m$, thus the rule $E\tau < 1$ is respected.)

The Newtonian binding energy of the concentric +/- shells is given, like in electrostatics, by the formula $E = (1/2) \int d^3x \rho(\mathbf{x}) V_{Newt.}(\mathbf{x})$. For general values $r_1$, $r_2$, $r_3$ of the radii and $\rho_1$, $\rho_2$ of the densities (constrained by the zero total mass condition (26)), one obtains a complicated expression, namely

$$\begin{aligned} E \;=\; & \frac{\pi \rho_1}{90 r_2^2} \{ \rho_2 (r_2^2 + r_2 r_3 + r_3^2)(6r_2^5 - 15 r_2^4 r_3 + 10 r_2^3 r_3^2 - r_3^5) - \rho_1 [9r_1^7 - 11 r_1^6 r_2 - r_1^5 r_2^2 - 10 r_1^4 r_2^3 + \\ & + 5 r_1^3 (r_2^4 + 2 r_2^3 r_3 + 2 r_2 r_3^3 - 2 r_3^4) + r_1^2 r_2^5 + r_1 r_2^6 + 2 r_2^3 (3 r_2^4 - 5 r_2^3 r_3 - 5 r_2 r_3^3 + 5 r_3^4)] \} \end{aligned} \tag{39}$$

We can study the sign and magnitude of $E$ setting $r_2 = r$, $r_1 = \alpha r$ $(0 < \alpha < 1)$ and $r_3 = \beta r$ $(\beta > 1)$. We express $\alpha$ in terms of $\beta$ using (26) and finally obtain

$$E = \frac{Gm}{r} P(\beta) \tag{40}$$

where $P(\beta)$ is a polynomial which is positive if $|\rho_1| > |\rho_2|$ (the repulsion between the two shells predominates) and negative if $|\rho_1| < |\rho_2|$ (the attraction inside each shell predominates).

This result is quite interesting, because

(i) Unlike the formula for the energy of the dipolar field, it does not contain any approximation to order $G$.

(ii) From the physical point of view it is reasonable to admit – remembering that we are in a weak-field regime and forgetting general covariance for a minute – that the binding energy is localized within the surface of the outer shell (the field is $o(G^2)$ outside). The energy density is therefore of the order of $\frac{|E|}{r^3} \sim \frac{Gm}{r^4} \sim 10^{29+k}\ cm^{-4}$ (with the parameters (19)), and can take both signs. This value looks quite large, even though the Ford-Roman inequalities [15] or similar bounds do not apply to quantum gravity, where the metric is not fixed but free to fluctuate, and there is in general no way to define a local energy density (except outside the sources – see [16]).

### 3.2 A scale-dependent $\Lambda$ ?

We have seen that a vacuum energy or cosmological term in the gravitational action is able to cut-off part of the dipolar fluctuations. This works better at large scales, because the $\Lambda$-term does not contain any field derivatives. We may also hypothesize that the effective value of $\Lambda$ at scales of the order of 1 $cm$ is larger than the value observed at cosmological scale. In the following we summarize some theoretical arguments supporting this idea. One would have, in other words, a small, negative, scale-dependent $\Lambda_{eff}$, a sort of residual of purely gravitational self-adjustment processes taking place at the Planck scale.

We already mentioned the role played by the cosmological constant at the classical level. In particular, looking for solutions of Einstein equations of the Friedman-Robertson-Walker type, i.e. with an expanding space, one finds well-defined relations between the Hubble constant, the density of various kinds of matter, and $\Lambda$ [9, 10]. In the last years, most estimates have given a *negative* value $\Lambda$ (in our conventions) of the order of $10^{-50}\ cm^{-2}$.

The effect of a cosmological term in the quantum field theory of gravity is less clear. On one hand, there are some "naive" expectations; on the other hand, formal results which are however difficult to interpretate.

The naive view consists in disregarding the effect of the cosmological term on the global geometry of spacetime, as compared to the effect of matter or pre-existing (null) curvature. Therefore one just expands the gravitational action around a flat background and studies quantum fluctuations. These are determined to leading order by the part of the action quadratic in $h_{\mu\nu}$. In spite of the different tensorial form of the Einstein term $\int dx\sqrt{g}R$ and the cosmological term $\int dx\sqrt{g}$, their quadratic parts are similar. In Feynman gauge they are both proportional to the quantity $[2\mathrm{Tr}h^2 - (\mathrm{Tr}h)^2]$, multiplied by $\partial^\mu\partial_\mu$ in the case of the Einstein term and by $\Lambda$ in the case of the cosmological term. Thus in this approximation the cosmological term corresponds to a mass term for the graviton; the mass is real for $\Lambda < 0$ and imaginary for $\Lambda > 0$ (in our conventions – see Section 1.1). This implies respectively a finite range propagator, á la Yukawa, with range of the order of $|\Lambda|^{-1/2}$, or the existence of unstable modes growing in time like real exponentials [17]. Intuitively, the reason for this behavior is clear (see also [18]), because a positive $\Lambda$ corresponds to a positive mass-energy density, which is gravitationally unstable.

In *pure quantum gravity* the curvature of the classical background is solely determined by $\Lambda$, and therefore the previous approach does not really make sense. For instance, if $\Lambda < 0$, then the solution of the classical Einstein equations is a spacetime with curvature radius of the order of $\Lambda^{-1/2}$; the Yukawa range predicted by the flat space expansion would then coincide with the size of the universe. There have thus been some attempts at quantizing the gravitational action with respect to a background with constant curvature (de Sitter or anti-de Sitter). The theory is mathematically very difficult [19]; there is some evidence, however, that the graviton stays massless, while novel strong infrared effects would arise (due to the dimensional self-coupling $\Lambda$), which might force the renormalized value of $\Lambda$ to "relax towards zero".

The *Euclidean theory of pure quantum gravity* is obtained from the Lorentzian theory in our conventions with the standard analytical continuation $t_{Lor} \to -it_{Eucl}$. In the lattice approach in 4D [20], $G$ and $\Lambda$ are entered as bare couplings at the beginning, and then the discretized space evolves according to a Montecarlo algorithm. Unlike in perturbation theory, where a flat background is introduced by hand, here flat space appears dynamically; namely, the average value of the curvature is found to vanish on a transition line in the bare-couplings space. This line separates a "smooth-phase", with small negative curvature, from a "rough", collapsed, unphysical phase, with large positive curvature. The collapse can be understood observing that the cosmological action is of the form $\Lambda V^{(4)}$, where $V^{(4)}$ is the volume of the lattice, thus when $\Lambda_{eff} = \langle R \rangle$ is positive, the volume tends to decrease.

It turns out that as the continuum limit is approached, the adimensional product $|\Lambda_{eff}|G_{eff}$ behaves like

$$|\Lambda_{eff}|G_{eff} \sim (l_0/l)^\gamma \tag{41}$$

where $l$ is the scale, $l_0$ is the lattice spacing, $\gamma$ a critical exponent and the sign of $\Lambda_{eff}$ is negative. Furthermore, one can reasonably assume that $l_0 \sim L_{Planck}$, and that the scale dependence of $G_{eff}$ is much weaker than that of $\Lambda_{eff}$.

A scale dependence of $\Lambda_{eff}$ like that in eq. (41) also implies that any bare value of $\Lambda$, expressing the energy density associated to the vacuum fluctuations of the quantum fields including the gravitational field itself, approaches zero at long distances just by virtue of the gravitational dynamics, without any need of a fine tuning. One would have, in other words, a purely gravitational solution of the cosmological constant problem.

It is remarkable that the conclusions of Euclidean lattice theory concerning the instability with $\Lambda > 0$ agree qualitatively with those obtained in the naive approach; and this in spite of the fact that the above argument concernig the volume of spacetime does not hold in the Lorentzian theory because in this case both positive and negative volume variations are suppressed by the oscillating factor $\exp[iS]$ in the functional integral.

### 3.3 Local changes in $\Lambda$

The ability of the $\Lambda$-term to cut-off part of the dipole fluctuations has an inevitable consequence. Consider the coupling of gravity to a scalar field $\phi$, with lagrangian density

$$L = \frac{1}{2}\left(\partial_\alpha\phi\partial^\alpha\phi - m^2\phi^2\right) = \frac{1}{2}\left[\left(\frac{\partial\phi}{\partial t}\right)^2 - (\mathrm{grad}\phi)^2 - m^2\phi^2\right] \tag{42}$$

and energy-momentum tensor

$$T_{\mu\nu} = \Pi_\mu\phi\partial_\nu\phi - g_{\mu\nu}L = \partial_\mu\partial_\nu\phi - g_{\mu\nu}L \tag{43}$$

The interaction term in the gravitational action is

$$S_{matter} = \frac{1}{2}\int d^4x\sqrt{g(x)}T^{\mu\nu}(x)h_{\mu\nu}(x) \tag{44}$$

and to lowest order in $h_{\mu\nu}$ we have

$$S_{matter} = \frac{1}{2}\int d^4x\left(h_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - \mathrm{Tr}hL\right) \tag{45}$$

On the other hand, the cosmological action is, still to lowest order in $h_{\mu\nu}$ and expanding $\sqrt{g} = 1 + \frac{1}{2}\mathrm{Tr}h + ...$

$$S_\Lambda = \frac{\Lambda}{8\pi G}\int d^4x\left(1 + \frac{1}{2}\mathrm{Tr}h\right) \tag{46}$$

We can say that to lowest order the coupling of gravity to the field $\phi$ produces a typical source term for $h_{\mu\nu}$, of the form $h_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$, and subtracts from the cosmological constant $\Lambda$ the local density $8\pi GL(x)$, because we can write, apart from an additive constant,

$$S_{matter} + S_\Lambda = \frac{1}{2}\int d^4x h_{\mu\nu}\partial^\mu\phi\partial^\nu\phi + \frac{1}{2}\int d^4x\mathrm{Tr}h\left(\frac{\Lambda}{8\pi G} - L\right) \tag{47}$$

This separation of the matter coupling in two parts looks in general quite arbitrary, but it can be useful if the lagrangian density is such to affect locally the "natural" cosmological term and set free gravitational fluctuations corresponding to virtual mass densities much larger than the real density of the field $\phi$.

An example will clarify our point. Suppose that $\phi$ represents some coherent fluid with the density of ordinary matter ($\sim 1\ g/cm^3$). We have seen that at the scale of 1 $cm$ the dipolar fluctuations are cut-off according to eq. (31). For $\Lambda$ equal to the cosmologically observed value of $\sim 10^{-50}\ cm^{-2}$, the exponent $k$ can take values up to $k = 3$, corresponding to fluctuations with virtual sources of density $\sim 10^3\ g/cm^3$. (This is a prudent estimate; for short-lasting fluctuations – less than 1 $s$ – and for those with $r_- = r_+$, the virtual mass density can be even higher.)

If the value of $L$ in some region is comparable to $\Lambda/8\pi G$, this can introduce an inhomogeneity in the cut-off mechanism. The result will be a local inhomogeneity of the dipolar fluctuations, which, given their strength, could dominate the effects of the coupling $h_{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ to the real matter.

Note that the magnitude of $L$ depends on whether $\phi$ is itself "on shell" or not. For a free, spatially homogeneous scalar field, for instance, the Klein-Gordon equation implies $\phi = const. \cdot e^{\pm imt}$. Therefore *on shell* one has $L = 0$ exactly, even though the single terms in $L$ can well be (for atomic-scale masses and gradients) of the order of $10^{33}\ cm^{-4}$.

The mechanism sketched above also has an Euclidean analogue [21], but a better understanding of the dipolar fluctuations is necessary before any progress in this direction can be made.

# 4 Conclusions

In the first part of this work we have studied the general features of "dipolar" zero modes of the pure Einstein action, giving some explicit examples in the weak-field approximation. We used a method based upon the classical Einstein equations with suitable virtual sources. Our aim was to prove in a rigorous way the null-action property of these modes. For applications to the quantum case we made reference to the (Lorentzian) functional integral. This represents just one of the possible approaches to quantum gravity, but in fact also the Planck-scale fluctuations have been studied through integral functional techniques [1, 11, 20]. It should be stressed that the numerical estimates presented in Section 2.2 are only lower limits based on specific examples. The strength of the fluctuations can be in general larger.

In the Discussion Section we have been less concerned with rigor. We have described some paradoxical features of the large dipole fluctuations, and possible suppression processes. The ADM energy of the dipolar fields can be both positive and negative, and turns out to be very large compared to their inverse duration $\tau^{-1}$. If we admit (as is quite reasonable for the +/- shells) that this energy is localized, the corresponding density appears large, too – even though the Ford-Roman inequalities or similar bounds do not apply to quantum gravity, where the metric is not fixed.

The hypothesis of a scale-dependent cosmological constant remains at present speculative, yet only the $\Lambda$-term seems to be capable of suppression at large scales. From the purely phenomenological point of view, the existence of a negative (in our conventions) $\Lambda_{eff}$, which reduces to the observed $\Lambda \sim 10^{-50}\ cm^{-2}$ at cosmological scale but is some orders of magnitude larger at $cm$ scale, is probably less disturbing than the existence of large quantum fluctuations [22].

Independently from the effective-$\Lambda$ hypothesis, the results of Sections 2.4 and 3.3 show that any local vacuum term of the form $g_{\mu\nu}(x)L(x)$ acts as a cutoff for the dipolar fluctuations, especially for those at large scale. This can cause local inhomogeneities, which are usually important when dealing with vacuum fluctuations in quantum field theory, and deserve further investigation.

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